The inversion-asymmetry of pion emission source along the outward direction in relativistic heavy ion collisions

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The inversion-asymmetry of the pion emission source in relativistic heavy ion collision under the Bertsch-Pratt convention is discussed and explicitly exhibited by a Monte Carlo model. The Gaussian source function popularly used in the HBT analysis of relativistic heavy ion collisions is invalid in this case. An inversion-asymmetric source function is suggested. A method for extracting the inversion-asymmetry degree of the source together with the source size from experimental data is proposed.

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The analysis of two-particle, in particular two-pion, correlation provides a powerful tool for determining the spatio-temporal characteristics of relativistic heavy ion collisions [1]. It is usually referred to as pion interferometry or HBT analysis, following the pioneer work of Hanbury-Brown and Twiss [2]. Unfortunately, there is no one-to-one correspondence between the two-particle correlation function in momentum space and the source distribution in configuration space. How to extract as much information as possible about the source distribution from the experimentally measured two-particle correlation data is the main challenge for the HBT analysis.

Two-particle correlation function can be expressed as

$$C(\boldsymbol{P},\boldsymbol{q}) = 1 + \int d^3\boldsymbol{r} S_{\boldsymbol{P}}(\boldsymbol{r}) |\phi_{\boldsymbol{q}}(\boldsymbol{r})|^2, \qquad (1)$$

where $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$ is the total momentum of the pair and $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$ is their relative momentum. $|\phi_{\boldsymbol{q}}(\boldsymbol{r})|^2$ is a weight factor, which takes care of the indistinguishability of identical particles. $S_{\boldsymbol{P}}(\boldsymbol{r})$ is the source function. The integral transform in Eq. (1) does not have a unique inverse due to the "on shell" constraint $\boldsymbol{P} \cdot \boldsymbol{q} = 0$ [3]. Therefore, to extract the distribution $S_{\boldsymbol{P}}(\boldsymbol{r})$ from the experimentally measurable correlation $C(\boldsymbol{P},\boldsymbol{q})$ additional assumption must be made on the functional form of the source function $S_{\boldsymbol{P}}(\boldsymbol{r})$, which should be consistent with the symmetry of the system.

Two kinds of symmetry are related to the present problem. The first kind is the rotational symmetry, *i.e.* the symmetry property with respect to the space rotation. For a central heavy ion collision there is axial symmetry about the beam axis, defined as *longitudinal direction*. This amounts to isotropic in the transverse plane, *i.e.* the source appears as a circle in the transverse plane with the same radius in all directions. For a non-central or peripheral collision, the axial symmetry is violated. The source has an ellipse form in the transverse plane. The radii along the two main axes are nonequal.

The second kind of symmetry is the inversion symmetry, *i.e.* the symmetry property with respect to the space inversion $x_i \to -x_i$. This symmetry holds for all the three coordinates in the laboratory frame. However, in the present relativistic heavy ion experiments the HBT analysis is carried out not in the laboratory frame but in the Bertsch-Pratt frame [4][5] — x_l, x_o, x_s, where the *longitudinal* component x_l still points along the beam direction; the *outward* component x_o is defined as parallel to the projection $\boldsymbol{K}$ of the pair momentum $\boldsymbol{P}$ on the transverse plane; the *sideward* component x_s is perpendicular to both x_l and x_o. This coordinate system has, in particular, the advantage that the ratio x_o/x_s is sensitive to the duration of hadronization [4][5][6]. It is essentially a pair-by-pair rotation of the laboratory frame so that the new coordinate x_o points to the pair transverse momentum $\boldsymbol{K}$ for all the pairs, *cf.* Fig. 1.

In the Bertsch-Pratt frame, the inversion transform of the coordinate x_o

$$x_o \to -x_o \qquad (2)$$

will change the direction of the vector $\boldsymbol{K}$, and therefore, is no longer a symmetry transform of the system. In other words, the existence of a fixed vector $\boldsymbol{K}$ along the direction x_o violates the inversion symmetry about $x_o = 0$, and a Gaussian source becomes invalid along this direction.

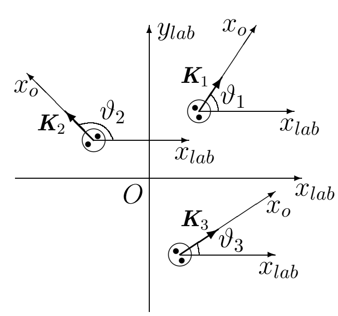

FIG. 1: The pair-by-pair rotation from laboratory frame to Bertsch-Pratt frame. In the figure ϑ_i is the rotation angle of $x_{\rm lab}$ for the ith pair.

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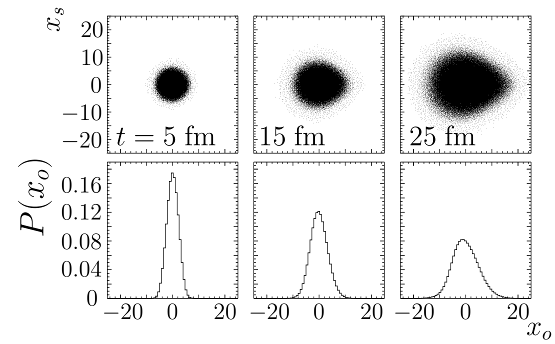

FIG. 2: Upper row: the x_o-x_s distribution of hadron pairs in a typical event of Au-Au central collision at $\sqrt{s_{\rm NN}} = 200$ GeV from AMPT simulation at three different time. From left to right $t = 5, 15, 25$ fm, respectively. The lower row is the projection to the x_o axis.

The aim of the present letter is to discuss how to take the inversion asymmetry along x_o into account in constructing a source function.

The inversion symmetry is an internal property of the distribution, which is defined in the intrinsic frame where the average value of coordinate vanishes,

$$\int_{-\infty}^{\infty} x\, P(x)\mathrm{d}x = 0. \tag{3}$$

Thus by definition a Gaussian is always inversion symmetric.

The *inversion-asymmetry degree* $\mathcal{A}$ can be defined in the above-mentioned frame as

$$\mathcal{A} = \sqrt{\frac{\int_{-\infty}^{\infty} \left[P(x) - P(-x)\right]^2 \mathrm{d}x}{2\int_{-\infty}^{\infty} [P(x)]^2 \mathrm{d}x}}\ , \tag{4}$$

which vanishes for inversion symmetric distribution and equals unity for maximum asymmetric distribution, where $P(x) \neq 0$ only when $P(-x) = 0$.

In current relativistic heavy ion experiments while performing HBT analysis a Gaussian source function is usually used. Such a function may be isotropic or anisotropic consistent with the rotational symmetric or asymmetric of the collision, but could never be inversion asymmetric.

As stated above, in relativistic heavy ion collisions the inversion symmetry holds for the source distribution in the laboratory frame, where the axes in the transverse plane are fixed and are common for all the particle pairs, but is broken along the x_o direction of the Bertsch-Pratt frame, *cf.* Fig. 1, so the Gaussian source function is no longer valid along this direction.

In order to illustrate how the x_o distribution violates the inversion symmetry and how the inversion-asymmetry develops in the heavy ion collision process, we take a Monte Carlo generator AMPT [7] as example. This model is based on non-equilibrium transport dynamics. It contains four main components: the initial conditions, partonic interactions, conversion from the partonic to the hadronic matter and hadronic interactions. The initial conditions, which includes the spatial and momentum distributions of minijet partons from hard processes and strings from soft processes, are obtained from the HIJING model [8] in which eikonized parton model is employed. The time evolution of partons is then treated according to the ZPC [9] parton cascade model. After partons stop interacting, a combined coalescence and string fragmentation model is used for the hadronization of partons. Scattering among the resulting hadrons are described by a relativistic transport (ART) model [10] which includes baryon-baryon, baryon-meson and meson-meson elastic and inelastic scattering.

In the upper row of Fig 2 is sketched the x_o-x_s distribution of hadron pairs in a typical event of $\sqrt{s_{\rm NN}} = 200$ GeV Au-Au central collision at three different times — $t = 5, 15, 25$ fm. The inversion-asymmetry along x_o can clearly be seen. The asymmetry degree develops stronger and stronger with the increase of time. The lower row is the projection to the x_o axis.

In order to extract the asymmetry information of source we need a source distribution function which can extrapolate from nearly symmetric, nearly Gaussian, to highly asymmetric. For this purpose we propose the following three-parameter N, B, a distribution

$$P(x) = \begin{cases} \frac{1}{B^{N+1}\Gamma(N+1)}(a \mp x)^N \mathrm{e}^{\frac{\pm x - a}{B}} & \pm x < a, \\ 0 & \pm x > a. \end{cases} \tag{5}$$

Transforming to the intrinsic frame, satisfying Eq. (3), we have

$$P_{NBa}(x) = \begin{cases} \mathrm{sgn}(B) \cdot \frac{[(N+1)B - x)]^N}{B^{N+1}\Gamma(N+1)}\ \mathrm{e}^{\frac{x-(N+1)B}{B}} & \text{for } \ \mathrm{sgn}(B)\cdot x < \mathrm{sgn}(B)\cdot (N+1)B, \\ 0 & \text{for } \ \mathrm{sgn}(B)\cdot x > \mathrm{sgn}(B)\cdot (N+1)B, \end{cases} \tag{6}$$

where

$$N > 0, \qquad \mathrm{sgn}(B) = \begin{cases} +1 & B > 0, \\ -1 & B < 0. \end{cases} \tag{7}$$

In Fig. 3 are plotted the $P_{NBa}(x)$'s for three different N. It can be seen that the distribution varies from ap-

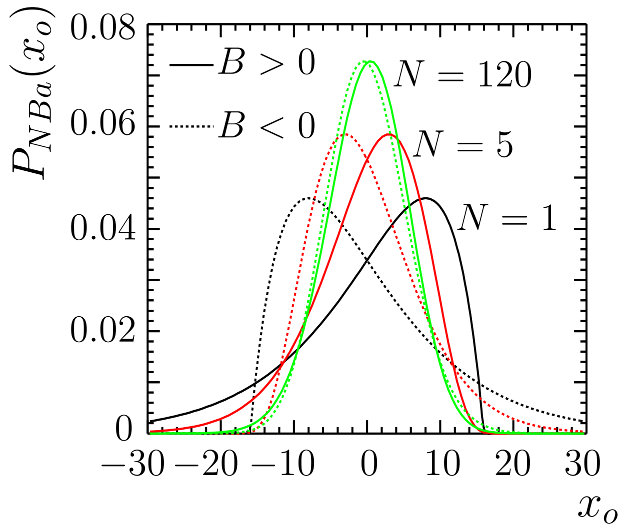

FIG. 3: (Color on line) The NBa distributions for 3 different values of N. The absolute values of B are adjusted to make the 3 curves have successively lower height.

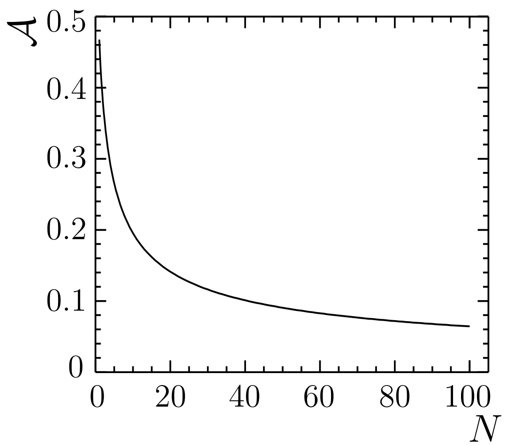

FIG. 4: The asymmetry degree of NBa distribution.

proximately symmetric about $x_o = 0$ at large N to highly asymmetric at small N. At sufficiently large N the NBa distribution Eq. (6) can mimic the Gaussian distribution very well.

It turns out that the asymmetry degree $\mathcal{A}$ of this distribution

$$\mathcal{A} = \sqrt{1 - \frac{\Gamma(\frac{1}{2})\Gamma(N+1)}{\Gamma(2N+1)\Gamma(N+\frac{3}{2})} \cdot [2(N+1)]^{2N+1} \mathrm{e}^{-2(N+1)}} \tag{8}$$

depends only on the parameter N but not on B and a. The asymmetry degree $\mathcal{A}$ as function of N is plotted in Fig. 4. It can be seen that $\mathcal{A}$ varies from ~ 0 at large N to ~ 0.5 at small N.

The size, or "radius", R defined as the root mean

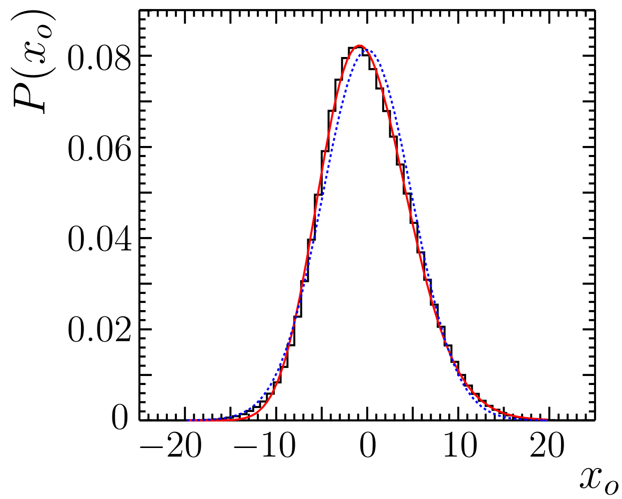

FIG. 5: (Color on line) The fit of the x_o distribution of the AMPT event at $t = 25$ fm used in Fig. 2 to NBa (solid line) and Gaussian (dashed line).

TABLE I: The size R_o and asymmetry degree $\mathcal{A}$ of a typical $\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au-Au central collision event at $t = 5, 15, 25$ fm used in Fig. 2, obtained from the fit of the x_o distribution to the NBa and Gaussian distributions, respectively

	Distribution			
	NBa		Gaussian	
Time	R_o	$\mathcal{A}$	R_o	$\mathcal{A}$
5 fm	2.28	0.0065	2.28	—
15 fm	3.30	0.0565	3.30	—
25 fm	4.90	0.1137	4.90	—

square of coordinate x is

$$R_{NBa} = \sqrt{\int_{-\infty}^{\infty} x^2 P_{NBa}(x)\mathrm{d}x} = |B| \cdot \sqrt{N+1}. \tag{9}$$

We fit the x_o distributions of the Monte Carlo events shown in the lower row of Fig. 2 to the NBa distribution (6) and present the results for $t = 25$ fm in Fig. 5 as solid line. For comparison the same distribution is also fitted to Gaussian and shown in the same figure as dashed line. It is evident that the NBa fit can account for the inversion-asymmetry of x_o distribution present in the Monte Carlo data, while the Gaussian fit fails to do so. The size R_o and asymmetry degree $\mathcal{A}$ resulting from the fit are listed in Table I. Using the NBa source function we are able to extract the asymmetry degree of the source, which could not be achieved using a Gaussian source function. As for the source size the results from both NBa and Gaussian source functions are the same.

The inversion asymmetry exists only in the outward direction, so we assume Gaussian sources in x_l and x_s,

while NBa source in x_o,

$$P(\boldsymbol{r}) \sim \mathrm{e}^{-\left[\frac{x_l^2}{2R_l^2}+\frac{x_s^2}{2R_s^2}\right]} \cdot P_{NBa}(x_o). \tag{10}$$

Performing Fourier transform we get

$$\rho(\boldsymbol{q}) \sim \frac{\mathrm{e}^{-\frac{1}{2}(q_l^2R_l^2+q_s^2R_s^2)}\mathrm{e}^{-iq_o(N+1)B}}{(1-iq_oB)^{N+1}}. \tag{11}$$

The correlation function is

$$\begin{aligned} C_2(\boldsymbol{q}) &= 1+\lambda|\rho(\boldsymbol{q})|^2 \\ &= 1+\lambda\frac{1}{[1+q_o^2B^2]^{(N+1)}}\,\mathrm{e}^{-(q_l^2R_l^2+q_s^2R_s^2)}. \end{aligned} \tag{12}$$

Through fitting the experimentally measured correlation function to Eq. (12) the parameters R_l, R_s, N and B are obtained. From these parameters beside being able to extract the size parameters R_l and R_s in the longitudinal and sideward directions, we are also armed with a powerful tool for extracting the asymmetry degree $\mathcal{A}$ and size parameter R_o in the outward direction by using Eq's. (8) and (9).

It is argued that under the Bertsch-Pratt convention [4][5] popularly used in the HBT analysis of relativistic heavy ion collision there is a fixed vector — the total transverse momentum $\boldsymbol{K}$ of the pair, along the outward direction. The existence of such a vector violates the inversion symmetry along this direction, making the conventional Gaussian source function invalid. Although the size of source can be extracted from the experimental data using a Gaussian source function, an important feature of the system — the asymmetry degree along x_o is missed.

In the present letter the inversion-asymmetry along x_o is pointed out in the first time. A 3-parameter source distribution —— the NBa distribution is proposed, which can extrapolate from nearly inversion-symmetric to highly inversion-asymmetric. Using a source function, which is Gaussian in the longitudinal and sideward directions, while NBa in the outward direction, the asymmetry degree $\mathcal{A}$ along the outward direction can be extracted from the experimental data together with the source-size parameters R_l, R_s and R_o.

The inversion asymmetry along x_o in a typical Au-Au central collision event under the Bertsch-Pratt convention has been checked using the AMPT Monte Carlo model, and the successful extraction of the asymmetry degree $\mathcal{A}$ from the data is exhibited. To apply the proposed method to real experimental data is highly recommended.

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